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On ionizing shock waves in a cylindrical plasma

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The 'switch-on' ionizing shock wave, initiated by a strong radial discharge between electrodes at one end of a cylindrical tube placed in a solenoid, moves along the tube with a speed that depends principally on the parameter

$$\xi = B_\theta B_0/(2e_0\rho_1),$$

where B_θ is the jump in the transverse magnetic field caused by the discharge current, B_0 is the applied axial field, ρ_1 is the upstream (neutral) gas density, and e_0 is the ionization energy. This simple result, established by Kunkel & Gross (1962) with the aid of the Chapman–Jouguet hypothesis and for fully ionizing shocks in an infinite-slab plasma, is here extended to cylindrical plasmas and to partially ionizing shocks. The effect of dissociation energy is also taken into account. The degree of ionization is determined by applying the Saha equation downstream of the shock. Most reported experiments fall into the partially ionized region, and are found to be in good agreement with the theory given in this paper.

1. Introduction

The production of plasmas by means of electromagnetically driven shock waves is now a familiar technique. A large number of field configurations and apparatus geometries have been used, and the appropriate shock equations have been developed for many of these. A particularly attractive and simple type of ionizing shock is that initiated by a strong radial discharge between electrodes at one end of a cylindrical tube situated in a solenoid (see figure 1). A shock front, more or less orthogonal to the tube axis, advances into neutral gas, in which region the applied magnetic field (due to the solenoid) is entirely axial. The shock front carries a strong radial current, so that downstream of the front the magnetic field will have an azimuthal component. This is an important example of 'switch-on' ionizing waves, which have been investigated experimentally by several people, notably Wilcox *et al.* (1960, 1962) and Brennan, Brown, Millar & Watson-Munro (1963). The essential features of these waves are that the direction of propagation is parallel to the undisturbed, upstream magnetic field and that the upstream gas is non-conducting. Another important feature, common to all ionizing waves, is the relatively large amount of energy that must be invested in ionization.

Many contributions to the general theory of ionizing shocks, which include switch-on shocks as a special case, have appeared in the literature recently. The geometry chosen is almost always an infinite slab (i.e. an 'annulus' of infinite radius), and the studies are most frequently concerned with the dynamics of fronts moving at right angles to the applied fields (e.g. see Kulikovski & Lyubimov 1960, Helliwell 1963, and Chu 1964). The theoretical studies most appropriate to the experimental arrangement of figure 1 are those of Kunkel & Gross (1962) and the extensions of their theory by Taussig (1964*a*, *b*); these contributions will be discussed shortly.

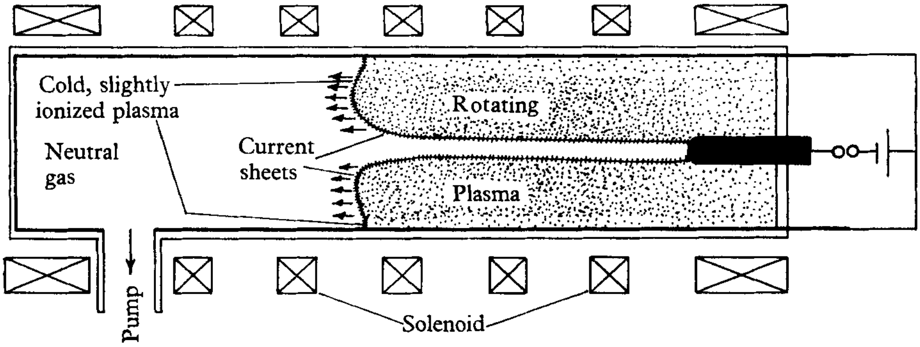

FIGURE 1. Schematic diagram of apparatus.

Perhaps the central problem of the theory of ionizing shock waves is that of finding a closed expression for the speed of the shock, u_1 relative to the upstream gas, in terms of some measure of the shock strength and the known upstream values of axial magnetic field B_0, density ρ_1 and pressure p_1. The conservation laws and Maxwell's equations are alone insufficient for this purpose, and just as with gaseous detonations, one further relationship is required in order that u_1 be determinable. When the upstream gas is pre-ionized, Ohm's law is applicable in region 1 (see figure 2) and provides the additional relation. This is usually taken to be the vanishing of the transverse component of the electric field $\mathbf{E}_1$ in the frame of the medium, a result that follows immediately from Ohm's law and the uniformity of the parallel velocity and magnetic fields, and the various shocks that can exist under these conditions have been studied by Bazer & Ericson (1960). When the upstream gas is neutral, or at most only slightly pre-ionized by diffusion of electrons and by radiation, E_r is not necessarily zero. For this case Kunkel & Gross adopted a modification of the Chapman–Jouguet hypothesis, familiar in gaseous detonations, and according to which the gas leaves the shock moving at sonic speed c_s, i.e. $u_2 = c_{s2}$. However, they did not allow for the possibility that the shocks may only partially ionize the gas, and confined their attention to the two-dimensional infinite-slab geometry.

Taussig in his first report makes a careful study of the stability of ionizing shocks moving parallel to a magnetic field in slab geometry, but does not use a closing relation and is thus forced to consider the whole class of ionizing shocks defined by the conservation laws and Maxwell's relations applied across the discontinuity. While interesting and useful, this study omits the very important energy losses due to dissociation and ionization. This is remedied in Taussig's second report, but the calculations are so extensive that he has confined attention to just two values of the upstream electric field, which takes a parametric role in the absence of a closing relation, and to hydrogen gas. He allows for the presence of four chemical species H_2, H, H^+ and e^- in chemical equilibrium in regions 1 and 2 (upstream and downstream of the shock) and provides computer solutions for the highly non-linear equations that result. However, unless the closing relation is determined, either by solving the very difficult problem of the shock structure or by accepting the C–J hypothesis, this work on partially ionizing shocks, while valuable, cannot be used to predict shock speeds.

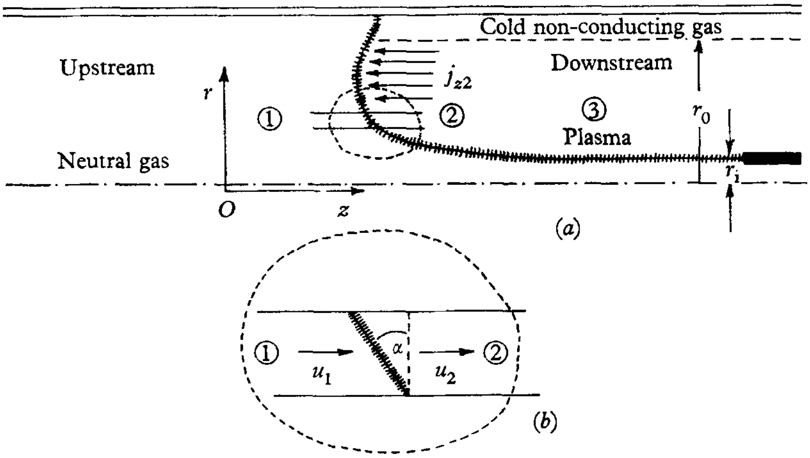

FIGURE 2. Theoretical model of shock.

What renders the shock structure problem so difficult—and certainly beyond analytical methods—is the fact that in one narrow region the flow goes through several distinct régimes of plasma physics, so that the transport coefficients have very complicated temperature and magnetic field dependence. Rosciszewski (1963) has made an effort to calculate the structure in argon gas, but the computing work required is not completed, and the model used, in assuming Saha equilibrium throughout the shock, is restricted to high-density plasmas.

Let b denote the ratio of the jump in the transverse magnetic field to the field normal to the shock front, then, as Kunkel & Gross (1962) have made clear, the C–J hypothesis is physically very reasonable for those shocks having $b \ll 1$. Clearly, if it is a steady-state solution that we seek, then we can permit neither a rarefaction wave to overtake the shock, nor a compressive wave to fall behind. The existence or not of a rarefaction wave depends, of course, on the boundary conditions at the end of the tube, say $z = 0$, from which the shock is receding. With a closed stationary end at $z = 0$, a rarefaction wave will pursue the shock, which will be time-independent only if $u_2 = c_2$, where c_2 is the speed of a magnetosonic expansion in the region immediately behind the shock. Such expansions propagate at the slower of the two distinct speeds available to magnetosonic waves.

Some doubt was cast on the applicability of the C–J hypothesis when the Kunkel–Gross theory failed to give agreement with the Australian experiments (Brennan *et al.* 1963). But it appears that until a large range of solutions are available for the shock structure we must rely on this hypothesis, and, as will be shown later in this paper, when the theory is modified to make allowance for (i) cylindrical geometry, (ii) dissociation and (iii) partial ionization, there is good agreement between theory and experiment. Our treatment avoids the complexities of Taussig's theory by the adoption of the Saha equation in region 2.

2. The mathematical model

Figure 2 (*a*) shows the mathematical model of the shock front we shall adopt. Region 1 is the upstream, undisturbed flow, which in a frame fixed in the front, has a velocity $u_1\hat{\mathbf{z}}$ and a magnetic field $B_{z1}\hat{\mathbf{z}}$, where $\hat{\mathbf{z}}$ is the unit vector parallel to the Oz-axis. Region 2 is a region of non-uniform flow immediately behind the shock—non-uniform in the sense that the curvature of the shock front induces some z-dependence on pressure, density and magnetic-field values. It is followed immediately by the expansion wave in region 3. For a reason to be made clear shortly, a volume current j_{z2} is assumed to flow from the plasma into the back of the shock front over the annulus $r_i < r < r_0$, where r_i is the radius at which the inner current sheet forms and r_0 is the effective outer radius of the plasma. The outer radius will be smaller than the tube radius due to the cold, unionized gas near the walls.

Figure 2 (*b*) shows a small section of the annulus $(r, r+dr)$ at which the shock is inclined at an angle $\frac{1}{2}\pi-\alpha$ to the tube axis. Unit normal to the shock is

$$\mathbf{n} = \hat{\mathbf{z}}\cos\alpha+\hat{\mathbf{r}}\sin\alpha,$$

and, if $\mathbf{j}^*$ is the sheet current in the shock, Maxwell's equations yield

$$\mathbf{n}\times[\mathbf{B}] = \mu\mathbf{j}^*, \quad \mathbf{n}\times[\mathbf{E}] = 0, \quad \text{and} \quad \mathbf{n}.[\mathbf{B}] = 0,$$

where $[X] \equiv X_2-X_1$. The equations involving $\mathbf{B}$ give

$$\left.\begin{aligned}[B_\theta] = B_{\theta 2} = -\mu J^*,\\ [B_z] = -\mu j_\theta^*\sin\alpha, \quad [B_r] = B_{r2} = \mu j_\theta^*\cos\alpha,\end{aligned}\right\} \tag{1}$$

where
$$J^* \equiv j_r^*\sec\alpha = -j_z^*\operatorname{cosec}\alpha,$$
so that
$$\mathbf{j}^* = J^*(\hat{\mathbf{r}}\cos\alpha-\hat{\mathbf{z}}\sin\alpha)+j_\theta^*\hat{\boldsymbol{\theta}}.$$

In these equations J^* defines the shock strength and can be assumed known (see equation (3)), whereas j_θ^* and α are unknown functions of radius that can be found only by solving the flow equations in regions 2 and 3. This is the most difficult problem arising from the cylindrical geometry. It can be avoided by assuming the front to be orthogonal to the tube axis at all points ($\alpha = 0$), in which case region 2 becomes a vanishingly small disc separating the shock from the expansion wave in region 3. Alternately if the region of the shock for which $\alpha \approx 0$ is fairly close to the wall, then the presence of the wall current sheet makes $B_{r2} \approx 0$, so that $j_\theta^* \approx 0$. And now by studying the shock relations across this *normal* section of the shock front, we are able to simplify considerably the calculations of the front velocity $-u_1$. As we are seeking an equilibrium solution, all other points on the front must have the same velocity relative to the upstream gas as the normal section. In the rest of this paper, then, we shall be dealing with the jump conditions for the small annular region in which α and j_θ^* are negligibly small.

Conservation of charge gives exactly

$$\frac{1}{r}\frac{\partial}{\partial r}(rJ^* \cos\alpha) - \frac{\partial}{\partial z}(J^* \sin\alpha) + j_{z2}\cos\alpha = 0 \tag{2}$$

on allowing for the presence of the feed current j_{z2}. Now it is found that the shock speed u_1 into the upstream gas depends directly on $B_{\theta 2}$ (see (15) and figure 4) and, as this speed is the same for all points on the shock, we conclude from (1) that at least for the region of small α, J^* must be constant. Hence by (1) and (2)

$$j_{z2} = -J^*/r = B_{\theta 2}/\mu r.$$

Without this volume feed current J^* would have $1/r$ dependence, and the shock profile would be time dependent. The physical origin of j_{z2} can be argued as follows. The electrons in the shock consist not only of those that started at one extreme radius, r_i or r_0, but also of fresh electrons from the newly ionized gas arriving from region 1, and these electrons, together with the new ions, will contribute to J^* by moving in opposite directions as they pass through the radial electric field in the shock front. This tendency to charge separation must be off-set by a current flowing into the shock from the plasma from the rear, and this is the current j_{z2}. A local drop in the value of J^* would reduce $B_{\theta 2}$, and so reduce the shock speed at the same point. Because of the increased transit time of the freshly ionized particles, this would tend to produce a greater charge separation and thus result in a greater volume current, so restoring the balance. There will be an effective outer radius r_0 at which the volume current is checked by the rapidly increasing (with radius) resistivity; thereafter J^* will have $1/r$ dependence. The total current passing through the shock and back along the outer current sheet is $I = 2\pi r_0 J^*$, so

$$B_{\theta 2} = \mu|I|/(2\pi r_0). \tag{3}$$

If $B_{\theta 2}$ is to be found from current rather than magnetic-field measurements, the problem of estimating the effective radius r_0 arises. In MHD wave experiments it has been found that as much as 10% of the tube radius must be considered lost to the wall boundary layer. A further point is that the expansion wave following the shock will result in a further increase in $B_{\theta 2}$ and will therefore carry some current. However this will be negligibly small if the magnetic pressure greatly exceeds the gas pressure (see Kunkel & Gross 1962, equation (75)). Thus I in equation (3) is the total current passing through the electrodes only if $c_A \gg c_S$, where c_A, c_S are the Alfvén and sound speeds defined below in (13).

It must be admitted that the question of the radial dependence of $B_{\theta 2}$ and other variables behind the shock is a difficult one, and such experimental results that are available are not conclusive on this point as they apply to the plasma some distance behind the front, and therefore in a different flow régime (see Sharp & Watson-Munro 1964). However these experiments do show that the front velocity is almost constant across the radius.

Let B_0 and E_0 be constants then, from (1) and $\hat{\mathbf{z}} \times [\mathbf{E}] = 0$, it follows that

$$B_{z1} = B_{z2} = B_0, \quad E_{r1} = E_{r2} = E_0. \tag{4}$$

B_0 is assumed to be given and E_0 is to be determined by the theory.

3. The shock-wave equations

Let ρ, p, h and $\mathbf{v} = (0, w, u)$ be the density, pressure, enthalpy and velocity (in cylindrical co-ordinates) of the whole gas, then the usual gasdynamic conservation laws require the constancy of ρu, $\rho u\mathbf{v} + \hat{\mathbf{z}}(p + B^2/2\mu) - \hat{\mathbf{z}}.\mathbf{BB}/\mu$ and $\rho u(\frac{1}{2}|\mathbf{v}|^2 + h) + \hat{\mathbf{z}}.\mathbf{E}\times\mathbf{B}/\mu$ across the shock, which is assumed to be sufficiently wide to allow heat flux to be neglected. Now set

$$u_2 = u_1 - v_2 \tag{5}$$

so that v_2 represents the inflow velocity of the ionized gas into the moving shock front, and use the axial component of the momentum-conservation equation to eliminate $u_1 v_2$ from the conservation-of-energy relation. As w_1 and $B_{\theta 1}$ are both zero, the outcome of this calculation is

$$\rho_1 u_1(e_2 - e_1 + \tfrac{1}{2}v_2^2 + \tfrac{1}{2}w_2^2) - u_1 B_{\theta 2}^2/2\mu = p_2 v_2 - E_0 B_{\theta 2}/\mu, \tag{6}$$

where e_1 and e_2 are the internal energies

$$e_1 = p_1/\{\rho_1(\gamma_1 - 1)\}, \quad e_2 = \tfrac{3}{2}(1+\mathscr{I})\,kT_2/m_i + (1-\mathscr{I})\,e_d + \mathscr{I}\,e_0, \tag{7}$$

in which e_d, e_0 are the dissociation and total ionization energies (starting with diatomic gas), m_i is the ion mass and $\mathscr{I}$ is the downstream degree of ionization. Here we have assumed that downstream of the shock the gas is wholly monatomic and that all particles are in thermal equilibrium.

The other conservation laws yield

$$\rho_2/\rho_1 = u_1/(u_1 - v_2), \tag{8}$$

and

$$\left.\begin{aligned} \rho_1 u_1 v_2 &= p_2 - p_1 + B_{\theta 2}^2/2\mu, \\ \rho_1 u_1 w_2 &= B_{\theta 2} B_0/\mu, \end{aligned}\right\} \tag{9}$$

where the downstream pressure is related to the temperature by

$$p_2 = (1+\mathscr{I})\,\rho_2 kT_2/m_i. \tag{10}$$

The radial component of Ohm's law in region 2, plus the assumption that $B_{\theta 2}$ is constant, i.e. $j_{r2} = 0$, leads to

$$E_0 = u_1 B_{\theta 2} - v_2 B_{\theta 2} - w_2 B_0. \tag{11}$$

Two difficulties face us in the application of the C–J hypothesis, viz. we need expressions for the phase velocities of magnetosonic waves in a rotating plasma with a steady-state helical field, and as these waves are dispersive, being subject to relaxation phenomena, we must specify their effective frequency, ω_p say. The calculation of the phase velocities is not at all easy unless attention is confined to the case of small helical angle, i.e. when

$$b \ll 1, \quad b \equiv B_{\theta 2}/B_0. \tag{12}$$

(Practically all the experimental results available to the author satisfy this condition, with b usually less than 0·05.) If it is further assumed that the magnetic and gas-pressure waves following the shock have negligible radial dependence, conforming with the assumption already introduced about the shock strength J^*, then all radial wave-numbers vanish. In this case the two waves reduce to the purely torsional Alfvén wave, and the purely longitudinal sound wave having velocities c_A, c_S given by

$$c_A^2 = B_0^2/\mu\rho_2, \quad c_S^2 = \gamma_2 p_2/\rho_2, \tag{13}$$

where in general γ_2 depends on ω_p, T_2 and $\mathscr{I}$, and lies between 1 and $\frac{5}{3}$ (e.g see Goldsworthy 1961). A Fourier analysis of the solitary wave spanning the rear end of the shock to the beginning of the expansion wave would entail *some* high frequency components at least, suggesting that the frozen-in value $\gamma_2 = \frac{5}{3}$ is the correct one to choose for equilibrium. However, as our final results are not very sensitive to the value selected for γ_2, we shall leave this point open. In the experiments the axial-field strengths are usually large enough to make $c_A \gg c_S$, and so confining attention to this case—which permits us to use equation (3) to determine $B_{\theta 2}$—we now have from the C–J hypothesis

$$u_2 = u_1 - v_2 = (\gamma_2 p_2/\rho_2)^{\frac{1}{2}} \quad (1 < \gamma_2 \leqslant \tfrac{5}{3}). \tag{14}$$

Finally on assuming thermal equilibrium behind the shock, we can complete our set of equations by using the Saha equation in region 2 to relate $\mathscr{I}$, T_2 and ρ_2.

4. Non-dimensional form of the theory

The large value of the ionization energy e_0 makes it natural to use the velocity defined by

$$u_c \equiv (2e_0)^{\frac{1}{2}}$$

as the reference velocity. We shall adopt the notation

$$\left.\begin{array}{lll} & x \equiv u_1/u_c, & \xi \equiv B_{\theta 2}B_0/(\mu\rho_1 u_c^2) = |I|B_0/(2\pi r_0 \rho_1 u_c^2), \\ & \tau \equiv kT/(m_i u_c^2), & \pi \equiv p/(\frac{1}{2}\rho_1 u_c^2), \quad \epsilon \equiv e_d/e_0, \\ & X \equiv x^2, & Y \equiv u_1 v_2/u_c^2, \quad \eta \equiv \xi b, \quad c \equiv \frac{1}{2}(\eta - \pi_1), \\ \text{and} & & z \equiv \mathscr{I} + (1-\mathscr{I})\epsilon - \pi_1/(\gamma_1 - 1) + \eta. \end{array}\right\} \tag{15}$$

Then (8), (9) and (10) take the form

$$\frac{\rho_2}{\rho_1} = \frac{X}{X-Y}, \quad \pi_2 = 2(Y-c), \quad \frac{w_2}{u_c} = \frac{\xi}{x}, \tag{16}$$

and

$$X = \frac{Y(Y-c)}{Y-c-(1+\mathscr{I})\tau_2}, \tag{17}$$

on eliminating π_2 from the last equation. Similarly, when E_0, π_2, w_2 and e_2 are eliminated from (6), it can be written in the form

$$X = \frac{\xi^2 + 4Y^2 - 5Yc + 2\eta Y}{z + 3(Y-c)}. \tag{18}$$

Equations (14) and (17) give

$$X = (1+\gamma_2)\,Y - \gamma_2 c. \tag{19}$$

An iterative method of solving (16) to (19) plus the Saha equation is easily devised—the approximate theory now following provides a pattern that could be adopted.

Taking advantage of (12) and the fact that as the neutral gas is usually at room temperature, $\pi_1 \ll 1$, we can neglect η and c, and then find that

$$x^2 = (1+\gamma_2)\,Y = \{(1+\gamma_2)^2/\gamma_2\}(1+\mathscr{I})\,\tau_2, \tag{20}$$

$$\xi^2 = x^2\{\mathscr{I}+(1-\mathscr{I})\,\epsilon+(hx)^2\}, \quad h^2 \equiv (3\gamma_2-1)/(1+\gamma_2)^2, \tag{21}$$

$$\rho_2/\rho_1 = (1+\gamma_2)/\gamma_2. \tag{22}$$

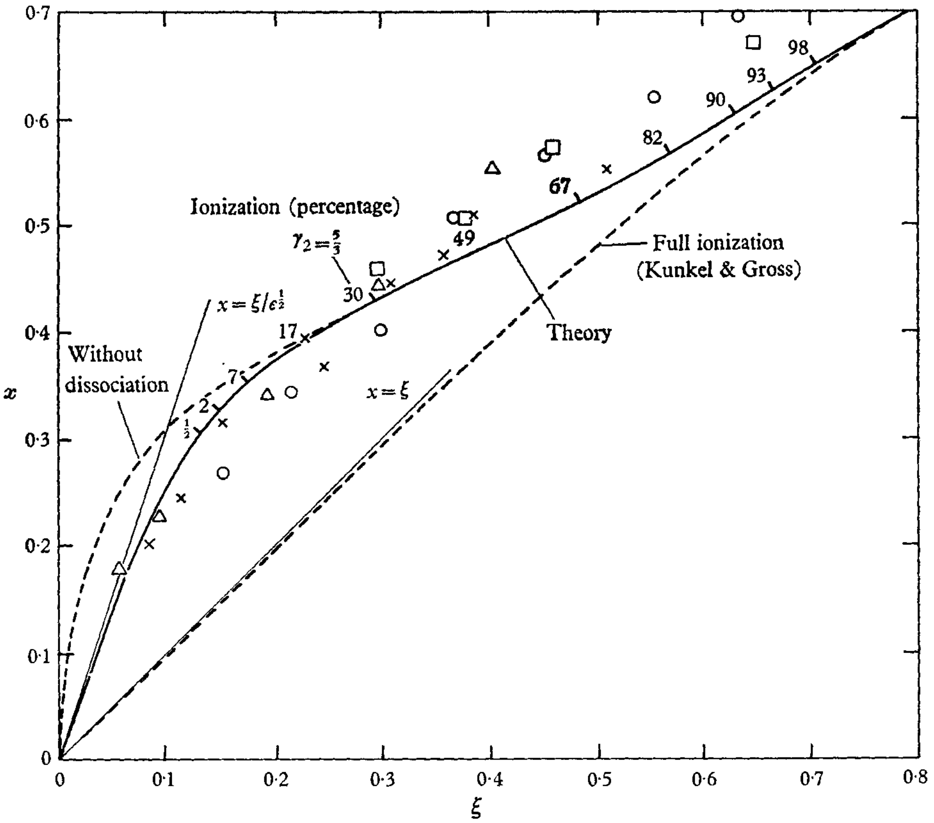

FIGURE 3. Comparison of experiment and theory for hydrogen and deuterium at $p_1 = 160\,\mu$Hg (curves for $\gamma_2 = 1$ and $\gamma_2 = \frac{5}{3}$ indistinguishable). Experimental points taken from Brennan *et al.*: △, H_2, 150 μHg, 15 kA; ×, D_2, 170 μHg, 15 kA; ○, D_2, 170 μHg, 7·7 kG; □, D_2, 7·7 kG, 15 kA.

For hydrogen or deuterium gas we can take the Saha equation in the approximate form

$$\log_{10}\left\{\frac{1-\mathscr{I}}{\mathscr{I}^2}\right\} = \frac{77620}{T_2} - \tfrac{3}{2}\log_{10}T_2 + \log_{10}\mathscr{P}_1 + \log_{10}\left(\frac{1+\gamma_2}{\gamma_2}\right) - 1\cdot 86, \tag{23}$$

with $\tau_2 = 2\cdot 75\times 10^{-6}T_2, \quad \epsilon = 0\cdot 117.$

In (23) $\mathscr{P}_1$ is the pressure expressed in μHg at 20 °C, and we have used (22) to eliminate ρ_2.

For the range $0\cdot 01 < \mathscr{I} < 0\cdot 99$ these equations are most easily solved as follows. Choose an upstream $\mathscr{P}_1$ and keep this fixed throughout the calculation.

Then choose T_2 and determine $\mathscr{I}$ from (23); then (20) and (21) yield the (x, ξ)-relationship. A succession of such calculations enables us to cover the $\mathscr{I}$ range quite quickly. Changes in $\mathscr{P}_1$, which usually lies in the range 20–1000 μHg, have very little effect on the shape of the $x(\xi)$ curve thus obtained.

For $\mathscr{I} > 0{\cdot}99$ we can neglect (23) and find

$$2\{(1+\gamma_2)^2/\gamma_2\}\tau_2 = x^2 = \frac{1}{2h^2}\{(1+(2h\xi)^2)^{\frac{1}{2}}-1\},$$

$$\approx \xi/h \quad \text{for} \quad \xi \gg 1, \tag{24}$$

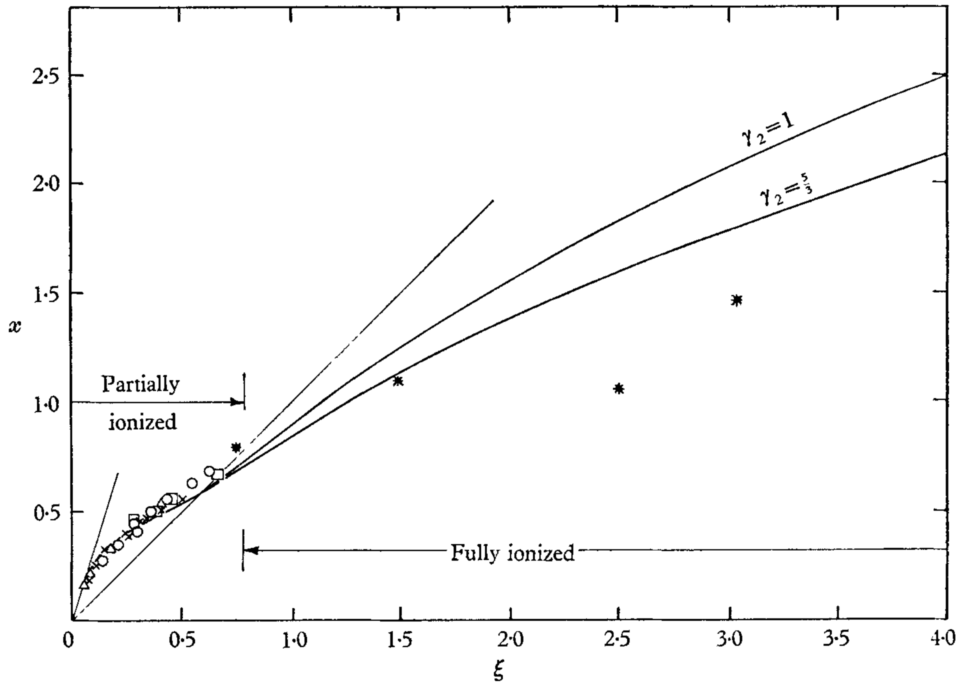

Figure 4. The fully ionized region. *, some unpublished low-pressure results.

while for $\mathscr{I} < 0{\cdot}01$,

$$\{(1+\gamma_2)^2/\gamma_2\}\tau_2 = x^2 = \frac{1}{2h^2}\{(\epsilon^2+(2h\xi)^2)^{\frac{1}{2}}-\epsilon\}, \tag{25}$$

$$\approx \xi^2/\epsilon \quad \text{near} \quad \xi = 0.$$

Figures 3 and 4 show on different scales the $x(\xi)$-relation for hydrogen and deuterium at $\mathscr{P}_1 = 160\,\mu$Hg. The eleven points shown in figure 3 on the $\gamma_2 = \frac{5}{3}$ curve for the indicated degrees of ionization start at $T_2 = 0{\cdot}8 \times 10^4\,^\circ$C (at $\mathscr{I} = \frac{1}{2}\,\%$) and increase by $10^3\,^\circ$C up to $T_2 = 1{\cdot}8 \times 10^4\,^\circ$C (at $\mathscr{I} = 98\,\%$). With $\gamma_2 = 1$ the $x(\xi)$ curve is hardly distinguishable from that shown in figure 3 for $\gamma_2 = \frac{5}{3}$. The relation between temperature and speed squares of the shock appears in figure 5.

5. Comparison of theory and experiment

The above theory is compared in figure 3 with the experimental results for shock speed given by Brennan *et al.* (1963). Other, as yet unpublished work by the Sydney University Department of Plasma Physics shows that, at the higher pressures at least, the shock speed is remarkably constant along the tube, strongly suggesting that a steady-state shock has been achieved. However, it should be noted that the experiments show that the radial discharge in the front occurs, not in a sheet as assumed in the theory, but in one, two or three spokes that wind up the tube in a helical path. It is difficult to estimate the effect this will have in producing some disagreement between theory and experiment.

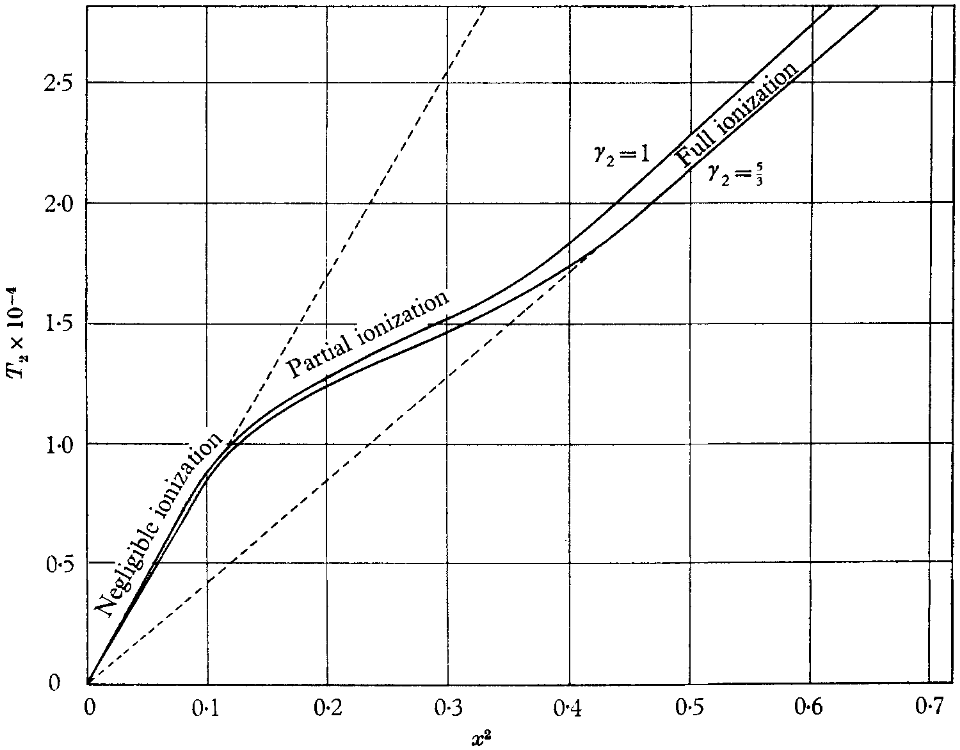

FIGURE 5. Temperatures in ionizing shocks.

As values of I rather than $B_{\theta 2}$ were given for the experiments, some estimate of the effective radius r_0 (see (15)) is required in order to determine ξ. Brennan *et al.* do give one measured value of $B_{\theta 2}$, namely 350 ± 50 G at $I = 15$ kA, from which we find that $r_0 = 8{\cdot}6$ cm in a tube of radius 10·25 cm. The points plotted in figure 3 are all based on this value of r_0, but even with a value of 10·25 cm there is substantial agreement between theory and experiment. Each experimental point carries quite an appreciable error bar, so that the scatter shown in the figure is to be expected. The importance of allowing for both dissociation and ionization is made clear by the figure.

In figure 4, most of which ($\xi > 0{\cdot}6$) is the régime that Kunkel & Gross considered, four additional experimental points are shown. These come from an unpublished report of the Sydney group, giving results for pressures down to $1\,\mu$Hg. They find that the velocity of the shock tends to constant values, independent of ρ_1 below $\mathscr{P}_1 = 20\,\mu$Hg. This deviation from the predictions of the theory could be explained, at least qualitatively, in several ways—e.g. (i) energy loss to walls, (ii) current shedding from the shock front so that $B_{\theta z}$ is no longer related to I by (3), (iii) thermal and/or dynamic equilibrium not achieved because of the short tube lengths involved, and (iv) viscous effects. Of these (ii) seems rather probable; again measurements of $B_{\theta 2}$ might resolve the problem.

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REFERENCES

Bazer, J. & Ericson, W. B. 1960 *Phys. Fluids*, **3**, 631.

Brennan, M. H., Brown, I. G., Millar, D. D. & Watson-Munro, C. N. 1963 *Plasma Phys.* (*J. Nucl. Energy*, Part C), **5**, 229.

Chu, C. K. 1964 *Phys. Fluids*, **7**, 1349.

Goldsworthy, F. A. 1961 *Prog. Aero. Sci.* **1**, 174.

Helliwell, J. B. 1963 *Phys. Fluids*, **6**, 1516.

Kulikovski, A. G. & Lyubimov, G. A. 1960 *Rev. Mod. Phys.* **32**, 977.

Kunkel, W. B. & Gross, R. A. 1962 *Plasma Hydrodynamics*, pp. 58–82. Stanford University Press.

Rosciszewski, J. 1963 *Space Sci. Lab. Rep., General Dynamics/Astronautics, San Diego, Calif.*, no. BBJ-63-002.

Sharp, L. & Watson-Munro, C. N. 1964 *Phys. Lett.* **11**, 39.

Taussig, R. 1964*a* *School Engng and App. Sci. Columbia University Rep.* no. 12.

Taussig, R. 1964*b* *School Engng and App. Sci. Columbia University Rep.* no. 14.

Wilcox, J. M., Baker, W. R., Boley, F. I., Cooper, W. S., DeSilva, A. W. & Spillman, G. R. 1962 *Plasma Phys.* (*J. Nucl. Energy*, Part C), **4**, 337.

Wilcox, J. M., Boley, F. I. & DeSilva, A. W. 1960 *Phys. Fluids*, **3**, 15.